

Answer

Chapter 3 Factors and Products - Notes

◆ 3.1 Factors and Multiples of Whole Numbers (p. 134)

• Prime and Composite Numbers - Vocabulary

→ **Factors** of a number: numbers that are multiplied together to get another number (the product)

→ **Product**: the number that results when two or more factors are multiplied.

Example: $2 \times 3 = 6$; the 2 and 3 are factors of 6, while 6 is the product.

You can arrive at the product of 16 by multiplying the following factors (numbers):

$$1 \times 16 = 16 \quad \text{or} \quad 2 \times 8 = 16 \quad \text{or} \quad 4 \times 4 \text{ or } 4^2 = 16$$

Therefore, the **factors of 16** are: {1, 2, 4, 8, and 16}.

→ **Prime number**: an integer greater than 1 that has only two different factors: number 1 and itself.

→ **Composite number**: an integer greater than 1 that has more than two factors.

Example: 2 is a prime number, since the only two factors are 1 and itself

4, 8 and 16 are composite numbers, since they all have more than two factors

not even
doesn't
and in
5

Circle all the **prime numbers** in the chart below.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49

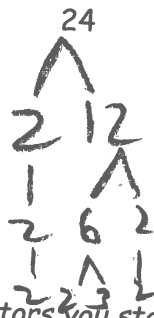
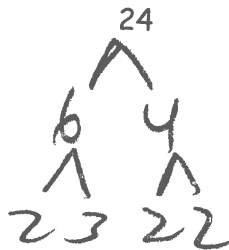
Notice that the numbers 0 and 1 are neither prime nor composite

- The number 1 has only one factor or divisor, not two or more.
- Zero has an infinite number of divisors, since zero can be divided evenly by any value and would still equal zero. Also, you cannot multiply two different non-zero values and have a product of zero.

1. Prime Factorization - Determining the Prime Factors of a Whole Number (p. 135)

The **prime factorization** of a number is the number written as the **product of its prime factors**. Find **two** factors of the given value and write them as branches on a **factor tree**.

Example: Use a factor tree to find the **prime factorization** of 24 in three ways: start with 6×4 , 2×12 , 8×3 . Six, four, twelve, and eight are **composite numbers** that can be factored further.



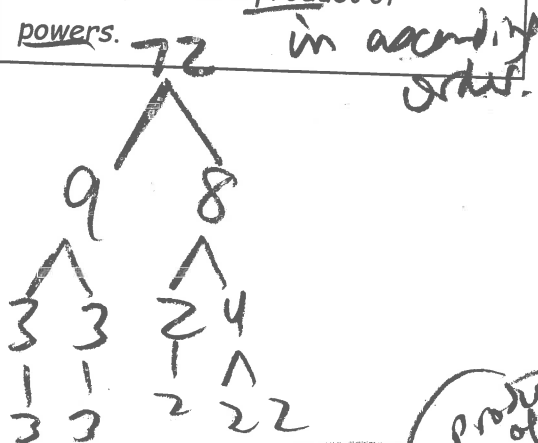
Note: No matter which two factors you started with, the prime factorization is the same.

There is only **ONE** correct **prime factorization** of a given value

Therefore,

- the **prime factors** of 24 are 2 and 3
- the **prime factorization** of 24 (written as the **product of its prime factors**) is: $2 \times 2 \times 2 \times 3$
- the **prime factorization** of 24 (written as the **product of powers**) is: $2^3 \times 3$

Example: Determine the **prime factorization** of 72, using a **factor tree**. Write the prime factorization **both** as a **product of its prime factors**, and as a **product of powers**.

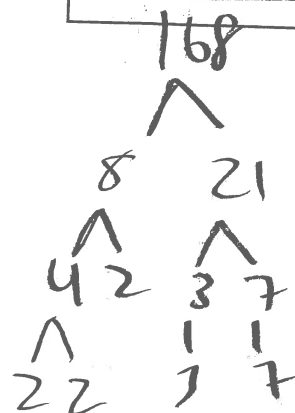


$$2 \times 2 \times 2 \times 3 \times 3$$

$$2^3 \times 3^2$$

product of prime factors
product of powers

Example: Write the **prime factorization** of 168. Write the prime factorization **both** as a **product of its prime factors**, and as a **product of powers**.



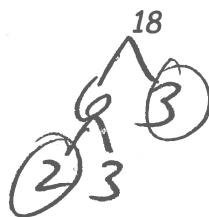
$$2 \times 2 \times 2 \times 3 \times 7$$

$$2^3 \times 3 \times 7$$

2. Determining the Greatest Common Factor (p. 136)

If **two or more** numbers have the **same** prime factor, it is called a **common factor**. The **greatest common factor** is the greatest factor that 2 or more terms have in common.

Example: Determine the **prime** factors of 30 and 18, using a factor tree. Highlight or circle the factors that appear in each prime factorization.

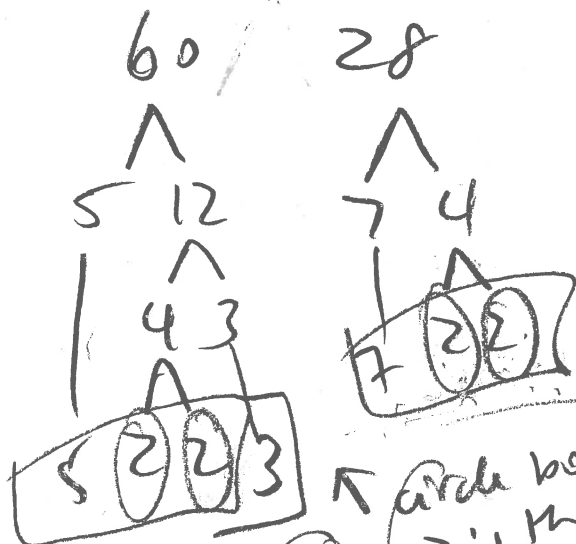


The **common factors** (the numbers you circled or highlighted) of 18 and 30 are 2 and 3. The **product** of these common prime factors is called the **greatest common factor**.

Therefore the **GCF** of 30 and 18 is 2×3 , or 6. The **GCF** is the **largest** number that divides two or more numbers. If two numbers have **no** common prime factor, the GCF is 1.

greatest factor the numbers have in common

Example: Determine the **greatest common factor** of 60 and 28

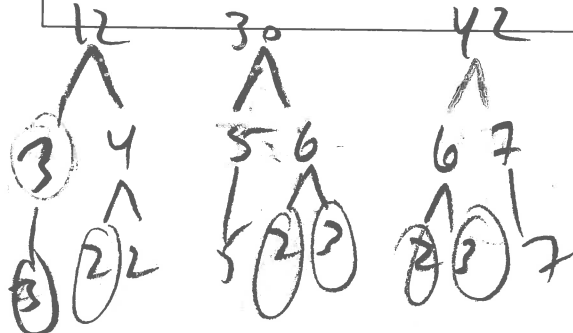


$$2 \times 2 = 4$$

GCF

circle both 2's that they have in common

Example: Determine the **greatest common factor** of 12, 30, and 42.



$$2 \times 3 = 6$$

GCF



3. Determining the Least Common Multiple (p. 137)

To determine the multiples of a number, multiply the number by the natural numbers (1,2,3,4..) For example, the multiples of 26 are 26, 52, 78, 104....

For 2 or more natural numbers, their Least Common Multiple is the smallest number that is a multiple of both or all the numbers. (The Least Common Multiple is the number we use for the common denominator when adding or subtracting fractions). In other words, it is the least number that is divisible by each number.

Example: Determine the Least Common Multiple of 18, 20, and 30

(Draw the factor tree of each number in order to find the prime factorization of each number as a product of powers. Highlight or circle the greatest POWER of each prime factor in ~~each~~ ^{any} list.) The least common multiple is the product of the greatest power of each prime factor.

18

$$\begin{array}{c} \wedge \\ 9 \quad 2 \\ \wedge \quad | \\ 3 \quad 3 \quad 2 \end{array}$$

2×3^2

20

$$\begin{array}{c} \wedge \\ 4 \quad 5 \\ \wedge \\ 2 \quad 2 \end{array}$$

$2^2 \times 5$

30

$$\begin{array}{c} \wedge \\ 3 \quad 10 \\ \wedge \\ 3 \quad 5 \quad 2 \end{array}$$

$2 \times 3 \times 5$

greatest power of:

$$\begin{array}{l} 2 \rightarrow 2^2 \\ 3 \rightarrow 3^2 \\ 5 \rightarrow 5 \end{array}$$

$$LCM = 2^2 \times 3^2 \times 5 = 180$$

Try it! Determine the LCM of 28, 42, 63

(answer 252)

28

$$\begin{array}{c} \wedge \\ 7 \quad 4 \\ | \quad \wedge \\ 7 \quad 2 \quad 2 \end{array}$$

$2^2 \times 7$

42

$$\begin{array}{c} \wedge \\ 6 \quad 7 \\ \wedge \quad | \\ 2 \quad 3 \quad 7 \end{array}$$

$2 \times 3 \times 7$

63

$$\begin{array}{c} \wedge \\ 9 \quad 7 \\ \wedge \quad | \\ 3 \quad 3 \quad 7 \end{array}$$

$3^2 \times 7$

LCM

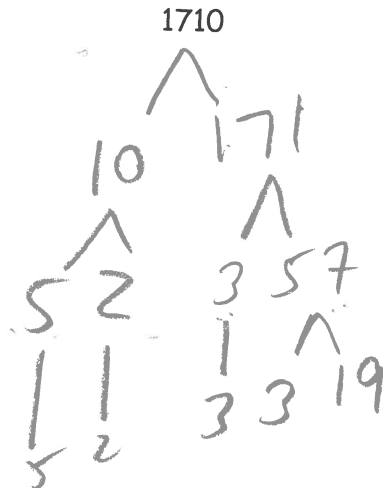
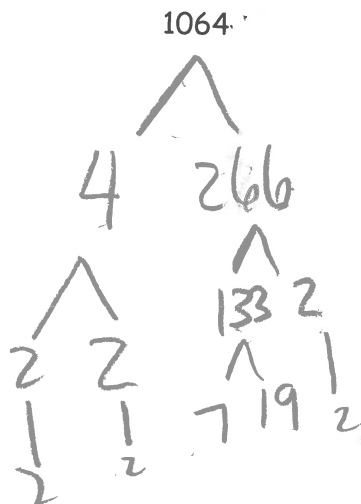
$$2^2 \times 3^2 \times 7 = 252$$

Y IT:

Now try finding the LCM and GCF of the same two numbers. For each, start with a factor tree to find the prime factorization. Write in ascending order.

To find GCF: Circle every factor that you find in BOTH lists (prime factors COMMON to BOTH). These can be repeated numbers - if there is a factor that appears more than once in EACH list. Multiply ALL common factors.

To find LCM: Write prime factorization as product of powers. Circle the greatest POWER of EVERY prime factor in ANY list.



Prime factorization in ascending order:

$$\underline{2 \times 2 \times 2 \times 7 \times 19}$$

$$\underline{2 \times 3 \times 3 \times 5 \times 19}$$

$$\text{GCF} = \underline{2 \times 19} = \underline{38}$$

(38)

Prime factorization as product of powers in ascending order:

$$\underline{2^3 \times 7 \times 19}$$

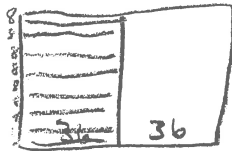
$$\underline{2 \times 3^2 \times 5 \times 19}$$

$$\text{LCM} = \underline{2^3 \times 3^2 \times 5 \times 7 \times 19} = \underline{47880}$$

(13 680)

4. Problem Solving using GCF and LCM (p. 138)

a) What is the side length of the smallest square that could be tiled with rectangles that measure 8 in by 36 in? Assume the rectangles cannot be cut.



$$\begin{array}{c} 8 \quad 36 \\ \wedge \quad \wedge \\ 4 \quad 9 \\ \wedge \quad \wedge \\ 2 \quad 3 \end{array}$$

The side length of the square must be a common multiple of 8 and 36.

Write the prime factorization of each number (as a product of powers).

Determine the LCM. *greatest prime of each*

$$LCM = 2^3 \cdot 3^2 = 8 \cdot 9 = 72$$

The smallest side of square would be 72 in.

b) What is the side length of the largest square that could be used to tile a rectangle that measures 8 in by 36 in? Assume that the squares cannot be cut. Sketch the square and rectangles.



$$\begin{array}{c} 8 \quad 36 \\ \wedge \quad \wedge \\ 4 \quad 9 \\ \wedge \quad \wedge \\ 2 \quad 3 \end{array}$$

The shorter side of the rectangle measures 8 in, so the side length of the square must be a factor of 8.

The longer side of the rectangle measures 36 in so the side length of the square must be a factor of 36.

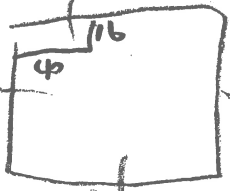
Side length of square must be common factor of 8 & 36. Write the prime factorization of each number.

Determine the GCF. *prime factors not occur in common*

Largest side of square would be 4 in.

Try "check your understanding" 4a, b p. 138

See p. 138, 139 for work example 4



$$\begin{array}{c} 40 \quad 16 \\ \wedge \quad \wedge \\ 4 \quad 10 \\ \wedge \quad \wedge \\ 2 \quad 5 \end{array}$$

a) 80 cm
b) 8 cm.

$$GCF = 2 \times 2 \times 2 = 8 \text{ cm}$$

$$LCM = 2^4 \times 5$$

HOMEWORK: 3.1 skill builder
p. 140 #5, 6, 8, 9 ~~not~~ Due tomorrow.

Information about 3.1 can be found on page 134-139 in your textbook
#7, #12

3.2 Perfect Squares, Perfect Cubes, and Their Roots

p. 142

Focus Find square roots of perfect squares and cube roots of perfect cubes.

1

A perfect square is the square of a whole number.

For example, 16 is a perfect square because $16 = 4^2$.

We say: 4 is the square root of 16.

We write: $\sqrt{16} = 4$.

100 is a perfect square.

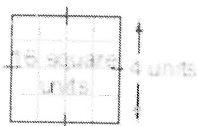
The prime factorization of 100 is:

$100 = 2 \times 2 \times 5 \times 5$, or $2^2 \times 5^2$

$= (2 \times 5) \times (2 \times 5)$ ↑ ↑
 $= 10 \times 10$ The prime factors occur in pairs.
 $\therefore \sqrt{100} = 10$

This is true for any perfect square.

So, we can use prime factorization to find the square root of a perfect square.



Area of square = length times width, but because length = width, we can say $A = s^2$ (s = length of side)

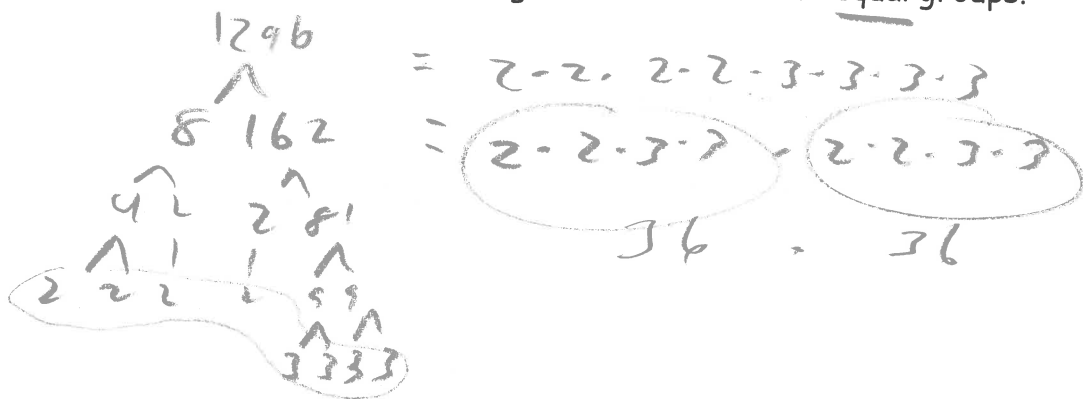
A **perfect square** is the product of 2 equal whole numbers. The square root of a number is one of the equal numbers.

A perfect square can be represented as the **area of s square** with a whole number side length.

The **square root of the area** of the square is the side length of the square.

Example: Write 1296 as the product of prime factors.

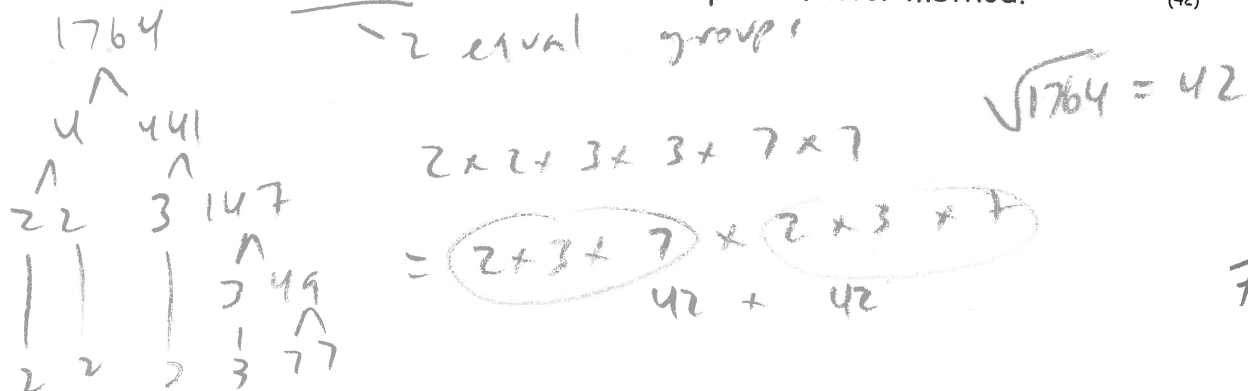
Group the factors in pairs. Rearrange the factors in two equal groups.



Since 1296 is the product of **two equal** whole numbers (36×36), its square root is **one** of these numbers. (In other words, 1296 is the area of the square, and $\sqrt{1296}$, or 36 is the side length of the square.) $\text{side}_{\text{square}} = \sqrt{\text{area of square}}$

Try it: Determine the square root of 1764 with the prime factor method.

(42)



2

A perfect cube is the cube of a whole number.

For example, 125 is a perfect cube because $125 = 5^3$.

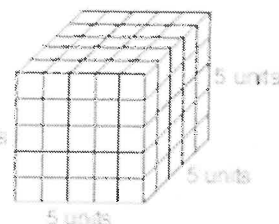
We say: 5 is the cube root of 125.

We write: $\sqrt[3]{125} = 5$

$$125 = 5 \times 5 \times 5$$

$$\therefore \sqrt[3]{125} = 5$$

Volume = 125 cubic units



$$V = \text{side} \times \text{side} \times \text{side} \text{ or } V = s^3$$

Example: Find $\sqrt[3]{1728}$, using prime factorization.

Group the factors in sets of 3. Rearrange the factors in **three** equal groups.

$$\begin{array}{r}
 1728 \\
 \wedge \\
 8 \quad 216 \\
 \wedge \quad \wedge \\
 24 \quad 8 \quad 27 \\
 \wedge \quad \wedge \quad \wedge \\
 2 \quad 2 \quad 4 \quad 3 \quad 3 \\
 \wedge \quad \wedge \quad \wedge \quad \wedge \quad \wedge \\
 2 \quad 2 \quad 2 \quad 2 \quad 3
 \end{array}$$

$$\begin{aligned}
 &2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \\
 &= (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 3) \cdot (2 \cdot 2 \cdot 3) \\
 &= 12 \cdot 12 \cdot 12 \\
 &\sqrt[3]{1728} = 12
 \end{aligned}$$

Since 1728 is the **product of 3 equal factors**, it can be represented by the **volume of a cube**. The side of the cube is equal to the cube root of 1728 (or the cube root of the volume). $1728 = 12 \times 12 \times 12$

so $\sqrt[3]{1728} = 12$

$$\text{edge}_{\text{cube}} = \sqrt[3]{\text{volume of cube}}$$

Try it: find the cube root of 2744, using prime factorization.

(14)

3 equal groups

$$\begin{array}{r}
 2744 \\
 \wedge \\
 4 \quad 686 \\
 \wedge \quad \wedge \\
 22 \quad 343 \quad 2 \\
 \wedge \quad \wedge \quad \wedge \\
 2 \quad 7 \quad 7 \quad 7 \quad 2
 \end{array}$$

$$\begin{aligned}
 &2 \cdot 2 \cdot 2 \cdot 7 \cdot 7 \cdot 7 \\
 &= (2 \cdot 7) \cdot (2 \cdot 7) \cdot (2 \cdot 7) \\
 &= 14 \cdot 14 \cdot 14 \\
 &\sqrt[3]{2744} = 14
 \end{aligned}$$

3. A cube has a volume of 4913 in^3 . What is the surface area of the cube?

(Remember if the volume is 4913, the side length is: $\sqrt[3]{4913} = 17$)

Also remember the formula for the surface area of a cube is: $6s^2$

(A cube has 6 faces. Each face is a square. The formula for area of a square is: s^2)

↑
side

L x W
s x s
s²

□ s
s

↑ side

$$\begin{array}{c} 4913 \\ \wedge \\ 17 \quad 289 \\ \wedge \\ 17 \quad 17 \\ 17 \cdot 17 \cdot 17 \end{array}$$

$$\sqrt[3]{4913} = 17$$

$$\text{Volume} = s^3 = 4913$$

$$\text{side of square} = \sqrt[3]{\text{Vol}} = \sqrt[3]{4913} = 17$$

(edge of cube)

↑
side

$$\begin{aligned} SA &= 6s^2 \\ &= 6(17)^2 \\ &= 6(289) \\ &= 1734 \text{ in}^2 \end{aligned}$$

← BEDMAS
↑ ↑
exp before mult

Homework: P. 146 # 4, 5 min 2
7a, 8a, 9

Polynomials - Review of Vocabulary and Operations

VOCABULARY

A **term** is a mathematical expression that can be a number, a variable, or the **product** of numbers and variables.

Examples of terms are: -5 or x or $5x$ or $-5xy$

A **polynomial** is a mathematical expression with one or more terms, formed by **adding or subtracting** terms.

Consider the polynomial: $-5x^2 + 4x - 2$

-It is made up of **three** terms formed by an addition and a subtraction operation.

-The **variable**, or unknown number, is represented by the letter, x .

-The exponents in the terms are given in **descending order of power**, such as x^2 followed by x^1 followed by x^0

-When a term is written without a variable, such as -2 , it is called a **constant**, because it will always have the same value.

-When a term has both a number and a variable, such as $-5x^2$ and $4x$, the number is called the **coefficient**, which tells you how many times to multiply the variable.

Polynomials can be named depending on the number of terms they have. Polynomial expressions with 1, 2, or 3 terms have special names:

Monomial - one term

Binomial - 2 terms

Trinomial - 3 terms

Polynomials with **more than** 3 terms are simply called polynomials.

Example: Complete the following chart.

Polynomial	# of terms	Name	Variables	Coefficients	Constants
$-x^2$	1	monomial	x	-1	0
$4y^3$	1	"	y	4	0
$5x^2 - 1$	2	binomial	x	5	-1
$8r^2 - 4r + 2$	3	trinomial	r	8, -4	2
$-6r^5 + 2r^3 - k - 10$	4	polynomial	r, k	$-6, 2, -1$	-10

OPERATIONS ON POLYNOMIALS - Review

Polynomials can be *combined* (added/subtracted) if they have *like terms* - two terms that have the same variable AND exponent, and differ *only* by the *numerical coefficient*.

Example: $17r^3t$ and $-4r^3t$ are *like terms*, because they have the same variables and exponents, r^3 and t^1 , and differ only by their coefficient, 17 and -4 .

Addition/Subtraction: to add or subtract like terms, you simply add/subtract the coefficients, and **keep the variables and exponents the same**.

To add polynomial expressions in brackets: if there is nothing or a "plus" in front of the brackets, remove the brackets **without changing the terms**.

Example: $(3x - 3y) + (4y - 2x)$

$$\begin{aligned}
 &= 3x - 3y + 4y - 2x && \leftarrow \text{(we remove the brackets without changing the terms)} \\
 &= 3x - 2x - 3y + 4y && \leftarrow \text{(we group like terms in descending order by degree and alphabetically)} \\
 &= x + y && \leftarrow \text{(Simplify by adding/subtracting like terms. if the coefficient is 1, we don't write the 1)}
 \end{aligned}$$

Try it : $(3x^2 - 5y) + (4y - 2x) + (-x^2 + 4x)$

$$\begin{aligned}
 &= 3x^2 - 5y + 4y - 2x - x^2 + 4x \\
 &= 3x^2 - x^2 - 2x + 4x - 5y + 4y \\
 &= 2x^2 + 2x - y
 \end{aligned}$$

To subtract polynomial expressions in brackets: you add the opposite.

Example: $(4m^2 - 2m - 4) - (-3m^2 - 2m + 5)$

When there is a subtraction or **negative sign in front of a bracket**, in the next step we don't write the **negative**, we remove the bracket, and we **change all the terms that were in the bracket to their opposite sign**. The rest of the steps are the same as addition (see above).

$$\begin{aligned}
 &(4m^2 - 2m - 4) - (-3m^2 - 2m + 5) \\
 &= 4m^2 - 2m - 4 + 3m^2 + 2m - 5 \\
 &= 4m^2 + 3m^2 - 2m + 2m - 4 - 5 \\
 &= 7m^2 - 9
 \end{aligned}$$

We don't change any of the terms in the first bracket because it had nothing or a positive in front of the bracket. We change all the terms in the second bracket to their **opposite signs** (and don't write the subtraction sign that was before the bracket).

Try it!

$$\begin{aligned}
 &: (-2x^2 + 7) - (3x^2 + x - 5) \\
 &= -2x^2 + 7 - 3x^2 - x + 5 \\
 &= -5x^2 - x + 12
 \end{aligned}$$

$$-5x^2 - x + 12$$

OPERATIONS ON POLYNOMIALS - Review, p. 2

Multiplication: to multiply polynomials, you apply *distribution* and the *exponent laws*.

→ To begin, you multiply the term outside the bracket by each term inside the brackets, separately.

Example: $2x(x^2 + 5x) = 2x^3 + 10x^2$

→ *Distribute* the term outside the bracket ($2x$) by multiplying it by the terms inside the bracket separately. To multiply the terms, first **multiply the coefficients**. Next, remember the **product law**. When multiplying variables, remember exponent laws. For the variables of the same base (same letter), **keep the base and add the exponents**.

Handwritten notes: $(2)(1)(x^1)(x^2) + (2)(5)(x^1)(x^1)$ add exponents, $2x^{1+2} + 10x^{1+1}$

Try it

Example: $-2(x + 5) = -2x + 10$

Example: $2x(x + 5x^2) = 2x^2 + 10x^3$

Division: to divide polynomials, you divide the coefficients and apply the *exponent laws*.

Example: $8x^4y^3 \div -2x^3y = -4x^1y^2$

Handwritten work: $(8) \div (-2) (x^{4-3}) (y^{3-1})$

→ Begin with dividing the coefficients.

→ Divide each variable separately. Remember the **quotient law**. When dividing the same base (same letter), **keep the base and subtract the exponents**. (When the exponent is "one", we don't write it.)

Handwritten note: think this

Try it: Simplify: $6x^3y^2z \div 12xy^2 = \frac{1}{2}x^2z$

Handwritten work:

$$\frac{6}{12} x^{3-1} y^{2-2} z = \frac{1}{2} x^2 y^0 z = \frac{1}{2} x^2 (1) z = \frac{1}{2} x^2 z$$

Handwritten note: think this

3.3 Exploring prime factorization and GCF with variables (p. 150)

Quick review: Find the prime factorization of: $20 = 2 \times 2 \times 5$



Question: How could you apply what you know about prime factorization to the term:

$$20x^2 = (2)(2)(5)(x)(x)$$

Quick review: Find the greatest common factor of 20 and 36. (Think of factors of 20 and 36.. the numbers that **divide into** 20 and 36. The GCF is the **LARGEST** factor **COMMON** to both. If you're not sure, you can do a factor tree.)

20: 1, 2, 4, 5, 10, 20 36: 1, 2, 3, 4, 9, 12, 18, 36 GCF = 4

Factor tree for 20: 20 → 4 × 5, 4 → 2 × 2. Factor tree for 36: 36 → 9 × 4, 9 → 3 × 3, 4 → 2 × 2.

GCF = $2 \times 2 = 4$

Question: Find the greatest common factor of the two terms.

$$GCF = 4x$$

$20x^2$ and $36x$

$(2)(2)(5)(x)(x)$ → $(2)(2)(3)(3)(x)$

Factoring Binomials - Greatest Common Factor

When a binomial is written as the product of its factors, the binomial has been factored. (To factor a polynomial, we write it as a product of its factors.) You can factor a polynomial using the greatest common factor method.

Example: Write each term of the binomial $4c^2 + 6c$ as a product of its factors, then circle the common factors.

$4c^2 = (2)(2)(c)(c)$ and $6c = (2)(3)(c)$ ∴ GCF = $2c$

Write each term as a product of the GCF and another monomial. (Divide the term by the GCF.)

Therefore, $4c^2 + 6c = 2c(2c) + 2c(3)$

Now write in factored form using distributive property to write the sum as a product.

$4c^2 + 6c = 2c(2c + 3)$

$\frac{4c^2}{2c} = 4 \div 2 (c^{2-1}) = 2c$
 $\frac{6c}{2c} = 3 c^0 = 3(1) = 3$

(We multiply $2c$ by what to get $4c^2$? Similarly, $2c$ times what equals $6c$? In other words, divide $4c^2$ by $2c$ and $6c$ by $2c$, remembering the quotient law where we **keep the base** and **subtract the exponents of the same variables**.)

Try it: Factor: $15n^2 + 6n$

$3n(5n + 2)$

GCF: $15n^2 \div 3n = 5n$, $6n \div 3n = 2$

$12s^2 + 3s$

$3s(4s + 1)$

GCF: $12s^2 \div 3s = 4s$, $3s \div 3s = 1$

Factoring and expanding are inverse processes. After you have factored, always quickly expand (on paper or in your head) again to check that you factored correctly.

Check by expanding:

$$2c(2c + 3) = \underline{4c^2 + 6c} \checkmark$$

$$3n(5n + 2) = \underline{15n^2 + 6n} \checkmark$$

$$3s(4s + 1) = \underline{12s^2 + 3s} \checkmark$$

Therefore, to verify that the solution to factoring is correct, you can use distributive multiplication.

In Arithmetic

Multiply factors to form a product

$$(4)(7) = 28$$

Factor a number by writing it as a product of its factors

$$28 = (4)(7)$$

In Algebra

Expand an expression to form a product

$$3(2 - 5a) = 6 - 15a$$

Factor a polynomial by writing it as a product of its factors

$$6 - 15a = 3(2 - 5a)$$

Try these: Factor completely. Check the factored form by expanding.

GCF (+)

GCF times first term in bracket equals first term of expression you're factoring

GCF times second term in bracket equals second term of expression you're factoring

a) $16x^2 - 12x$

$$4x(4x - 3)$$

check: $16x^2 - 12x \checkmark$

b) $49e^2 - 14e$

$$7e(7e - 2)$$

$49e^2 - 14e \checkmark$

c) $16h - 64h^2$

$$16h(1 - 4h)$$

$16h - 64h^2 \checkmark$

d) $12x^4 + 16x^2$

$$4x^2(3x^2 + 4)$$

$12x^4 + 16x^2 \checkmark$

Common factor when leading coefficient is negative

base -3 is squared \rightarrow Recall the difference between $(-3)^2$ and -3^2 . Write each as repeated multiplication.

$$\begin{aligned} (-3)(-3) &= 9 \\ - (3)(3) &= -9 \end{aligned}$$

This concept applies to factoring binomials with negative numbers.

If the first term of the polynomial is negative, we factor out the negative.

Example: Factor the binomial: $-16t^2 - 24t$

$$\begin{aligned} -16t^2 &= 1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot t \cdot t \\ -24t &= -1 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot t \end{aligned}$$

The GCF is $-8t$

It is important to note that the negative is -1 and needs to be included in the GCF.

As a rule, when the largest degree of a polynomial is a negative, factor it out.

$$-16t^2 - 24t = -8t(2t - 3)$$

What would happen if you did not include the negative, and the GCF was $8t$?

$$8t(-2t^2 - 3t)$$

\uparrow leading (first term) in bracket would be negative. $\ddot{\sim}$

Example: Factor the binomial: $-12n^2 + 8n$

In this example, -1 is not a common factor; however, because the largest degree is a negative, you still need to factor out the negative, and include it in the GCF.

$$-12n^2 + 8n = -4n(3n - 2)$$

It is important that the leading (first) term in the brackets is positive.

Factoring distribution:

$$-12n^2 + 8n$$

What would happen if you did not include the negative, and the GCF was $4n$?

leading term in bracket would be negative.

Homework: _____

2. Common Factors of Trinomials in the form $ax^2 + bx + c$ (p. 153)

You can factor a trinomial or any polynomial by using the greatest common factor method.

Example: Factor the trinomial: $5c^2 - 10c + 5$

$$5c^2 - 10c + 5$$

$$-10c = \underline{2(5)c} \quad \text{The GCF is } \underline{5}$$

Divide the trinomial by the greatest common factor:

$$(5c^2 - 10c + 5) \div 5 = \underline{c^2 - 2c + 1}$$

$$\frac{5c^2}{5} - \frac{10c}{5} + \frac{5}{5}$$

Write the polynomial in factored form, which is the greatest common factor multiplied by the quotient:

$$\underline{5(c^2 - 2c + 1)}$$

To find the GCF, think of the largest common factor of the coefficients. Then think of the largest power of the variable that is common to all terms.

Try it: Factor the trinomial then expand to check.

$$-18n^3 - 12n^2 + 6n$$

leading term is negative

factor out negative

$$-6n(3n^2 + 2n - 1)$$

$$-18n^3 - 12n^2 + 6n$$

↑ Write the polynomial in factored form, which is the greatest common factor multiplied by the quotient. ↑

When the leading coefficient is a negative, factor out the negative.

homework = p. 156
16 at least 9, 95

3. Common Factors of Polynomials in More than One Variable (p. 154)

Find the GCF of all the terms. Think of the largest common factor of the coefficients. Write the coefficient as a negative number as the leading coefficient is negative and you need to factor out the negative. Then think of the largest degree of the variable that is common to all terms.

Next, write the polynomial in factored form, which is the GCF multiplied by the quotient. (The quotient is the polynomial divided by the GCF.) Each term in the bracket, when multiplied by the GCF, gives the original term.

To find each term in the bracket think, " " times what equals the original term?
GCF

Example: Factor the trinomial then expand to check. $-12x^3y - 20xy^2 - 16x^2y^2$

$$-4xy (3x^2 + 5y + 4xy)$$

Figure out coefficient + each variable separately.
-4xy times — equals original term?

$$\text{Check} = 12x^3y - 20xy^2 + 16x^2y^2$$

Try it: Factor the trinomial then expand to check. $-20b^4c - 30b^3c^2 - 25bc$

$$-5bc (4b^3 + 6b^2c + 5)$$

$$\text{check} = -20b^4c - 30b^3c^2 - 25bc$$

HOMEWORK _____

3.5 Multiplying Two Polynomials: The Distributive Property and FOIL (p. 161)

-When we have a term multiplied by a polynomial, we multiply that term by **each** of the terms in the polynomial. (distribution)

Example: $2x(x + 5x^2 - 10x^3) =$ $2x^2 + 10x^3 - 20x^4$

1. When multiplying a **binomial** by a **binomial**, we multiply **each** term of the first binomial by **each** term of the second binomial.

Example: $(x + 2)(x + 4) =$ $x^2 + 4x + 2x + 8$
 $=$ $x^2 + 6x + 8$ (simplify by combining like terms)

$(x + 2)(x + 4) = x^2 + 6x + 8$ ← Notice that **8** is the **PRODUCT** of 4 and 2 and **6** is the **SUM** of 4 and 2.

An acronym that can be used to help you remember an order of distribution when multiplying a **binomial** by a **binomial** is **FOIL**, which stands for: **First Outside Inside Last**

Example: $(4x - 1)(x + 3)$

$4x^2 + 12x - x - 3$
 $= 4x^2 + 11x - 3$

- | | | | |
|----------|--|-----------|----------|
| F | Multiply the first term in each binomial | $(4x)(x)$ | F |
| O | Multiply the two outside terms in each binomial | $(4x)(3)$ | O |
| I | Multiply the two inside terms in each binomial | $(-1)(x)$ | I |
| L | Multiply the last term in each binomial | $(-1)(3)$ | L |
- Then simplify by combining like terms.

Example:

simplify $(x + 5)^2$
 $= (x + 5)(x + 5)$

$= x^2 + 5x + 5x + 25$
 $= x^2 + 10x + 25$

positive perfect square

We call these **perfect square trinomials**. (They are **trinomials** (3 terms), and the first and last terms are **perfect squares** (hence "**perfect square trinomial**"). The middle term is the **product** (multiplied by 2) of the first and last terms in the factored binomial. The middle term will be **added** if the factored form has a **plus**; or **subtracted** if the factored form has a **subtraction**. You can memorize this pattern to make it quicker to multiply.

minus
 Simplify $(x - 5)^2$
 $= (x - 5)(x - 5)$
 $= x^2 - 5x - 5x + 25$
 $= x^2 - 10x + 25$

negative perfect squares

Example: $(x + 5)(x - 5)$

$= x^2 - 5x + 5x - 25$
 $= x^2 - 25$

We call this a **difference of squares**. (It is a **binomial** → 2 terms. The first and last terms are **perfect squares**. The terms are **subtracted** – hence "difference" of squares.)

Multiply the following polynomials.

a) $(4a^2b)(5a^2b^2)$
 $= 20a^4b^3$

b) $(-\frac{3}{4}mn^2)(8m^3n^2)$
 $(-\frac{3}{4})(8)(m^4n^4)$
 $= -6m^4n^4$

c) $5a(a+7)$
 $5a^2 + 35a$

d) $(-2g+8)(7-3g)$
 $-14g + 6g^2 + 56 - 24g$
 $= 6g^2 - 38g + 56$
 write in descending
 order of degree (+ a, b, c, etc.)

e) $(y+3)(y+4)$
 $= y^2 + 4y + 3y + 12$
 $= y^2 + 7y + 12$

f) $(2a-5)^2 = (2a-5)(2a-5)$
 $= 2a^2 - 10a - 10a + 25$
 $= 2a^2 - 20a + 25$

Find the value when $m = -2$ and $n = 4$.

$m^2 + n(m-n)$
 $= (-2)^2 + 4(-2-4)$
 $= 4 + 4(-6)$
 $= 4 - 24 = -20$

or
 perfect square
 trinomial
 pattern:
 $(2a)^2 + 2(-5)(2a) + (-5)^2$
 $= 4a^2 - 20a + 25$

Solve the following equation.

$2(\frac{x}{2}) - (3)^2 = (x)^2 + (1)2$
 $x - 6 = x^2 + 2$
 $x = x^2 + 8$
 $-x^2 = 8$
 $\frac{-x^2}{-1} = \frac{8}{-1}$
 $x^2 = -8$
 $x = \pm \sqrt{-8}$

or $\frac{x}{2} - 3 = x + 1$
 $\frac{x}{2} = x + 4$
 $-\frac{x}{2} = 4$
 $\frac{1}{2}x - \frac{2}{2}x = 4$
 $-\frac{1}{2}x = 4$
 $-2(\frac{1}{2}x) = 4(-2)$
 $-x = -8$
 $x = 8$

2. Factoring Trinomials of the Form $ax^2 + bx + c$ ($a = 1$) – **PRODUCT/SUM method** (p. 163)

Recall:

$$(x+2)(x+4) = x^2 + 6x + 8$$

← Notice that 8 is the **PRODUCT** of 4 and 2 and 6 is the **SUM** of 4 and 2.

Since expanding (distributing) and factoring are inverse processes, **factoring the trinomial**, $x^2 + 6x + 8$, should result in, $(x+2)(x+4)$.

When the leading coefficient (a) is equal to 1, **always** use the **product sum method**. This method finds two factors whose **product** is equal to c , and whose **sum** is equal to b .

Example: Factor the trinomial: $x^2 + 10x + 24$

(The first step in factoring is **always** to see if there is a greatest common factor (GCF) (and if so, to factor using GCF method. However, when the **leading coefficient (a) is equal to 1**, there is **NO** greatest common factor.)

List the pairs of factors of 24: 1×24 ; 2×12 ; 3×8 ; 4×6

Think: Which of these pairs of numbers could possibly add or subtract to equal 10?

(You are looking for factors whose product equals +24, and whose sum equals +10.)

Factors of 24

1×24
 2×12
 3×8
 4×6

Sum of the Factors

25
24
11
10

Because factoring and expanding are inverse processes, factoring a trinomial will result in binomial multiplication, or $(x + \text{integer})(x + \text{integer})$. Therefore, write the factors as the **integers** in each of the binomials:

$$(x+4)(x+6)$$

Example: Factor the trinomials and verify using distribution (FOIL)

check: $x^2 - 12x + 35 = x^2 - 12x + 35$

a) $x^2 - 12x + 35$

$(x-5)(x-7)$

↑
make signs negative
 $-5-7 = -12$

1×35
 5×7

sum is 12

$(-5)(-7) = 35$

B) $a^2 + 7a - 18$

$(a+9)(a-2)$

$(9)(-2) = -18$
 $9-2 = 7$

C) $n^2 - 2n - 8$

$(n-4)(n+2)$

$(-4)(2) = -8$
 $-4+2 = -2$

20

NOTE: there are 2 rules that can help you decide which 2 factors to use

1. When c is negative, the two factors must have different signs (one positive and one negative). If b is positive, the greater factor is positive, while if b is negative, the greater factor is negative.
2. When c is positive, the two factors must have the same signs (2 positives or 2 negatives). When c is positive, b determines the sign of the two factors. If b is positive, the 2 factors are positive, and if b is negative, the two factors are negative.

Factoring Trinomials in Ascending Order

As the first step when factoring, check to make sure the polynomial is written in descending order (and if it is not, rewrite the expression in descending order).

Example: Factor the trinomial: $-24 - 5a + a^2$

Hint: Rewrite the polynomial in descending order, and then factor.

$$(a-8)(a+3) \quad a^2 - 5a - 24$$

Verify your answer:

$$\begin{aligned} & a^2 + 3a - 8a - 24 \\ & = a^2 - 5a - 24 \\ & = -24 - 5a + a^2 \end{aligned}$$

Example: Factor the trinomial: $16 - 6x - x^2$

$$(a-8)(a+2) \quad x^2 - 6x + 16$$

Verify your answer:

$$\begin{aligned} & x^2 + 2x - 8x - 16 \\ & x^2 - 6x - 16 \\ & -16 - 6x + x^2 \end{aligned}$$

HOMEWORK: _____

4. Factoring Trinomials with a Common Factor and Binomial Factors (p. 165)

Example: Factor the trinomial: $-m^2 + 18m - 80$

When factoring a trinomial in the form $ax^2 + bx + c$, where the coefficient (a) is **NOT** equal to 1, the first step is **always** to see if there is a greatest common factor (GCF), and if so to factor using the GCF method. (In this case, factor out -1 to make leading coefficient equal to 1.)

$$\begin{aligned} & -(m^2 - 18m + 80) \\ & = -(m - 8)(8 - 10) \end{aligned}$$

Example: Factor the trinomial: $-4t^2 - 16t + 128$

Find the greatest common factor first. If doing that changes the leading factor to 1, then factor the polynomial using sum and product method.

$$\begin{aligned} & = -4(t^2 + 4t - 32) \\ & = -4(t + 8)(t - 4) \end{aligned}$$

Since factoring and expanding are inverse processes, check that the factors are correct by distributing them. Remember to multiply by -4 at the very end.

$$\begin{aligned} & -4(t^2 + 4t + 8t - 32) \\ & -4(t^2 + 12t - 32) \\ & -4t^2 - 16t + 128 \checkmark \end{aligned}$$

Example: Factor the trinomial: $10x^2 + 80x + 120$

$$\begin{aligned} & 10(x^2 + 8x + 12) \\ & 10(x + 6)(x + 2) \end{aligned}$$

$$\begin{aligned} \text{Check} &= 10(x^2 + 2x + 6x + 12) \\ &= 10(x^2 + 8x + 12) \\ &= 10x^2 + 80x + 120 \end{aligned}$$

Homework: _____

3.6 Factoring Trinomials $ax^2 + bx + c$ ($a \neq 1$)

1st terms have coefficients
coefficients = 1

Expand and simplify the following: $(x-4)(x+2)$ and $(3d+4)(4d+2)$

$$x^2 + 2x - 4x + 8$$

$$x^2 - 2x + 8$$

Leading term =

$$12d^2 + 6d + 16d + 8$$

$$12d^2 + 22d + 8$$

Leading term $\neq 1$

When the leading coefficient (a) is *not* equal to 1, factoring takes a little bit more work. Here are a couple of methods to try:

- **Trial and error Method (reverse foil)** Factor $ax^2 + bx + c$ where $a \neq 1$. (p. 172)
 (Nail down your first and last terms and try different combinations to get your outer and inner terms to equal your middle term.)

WRITE LIGHTLY in PENCIL. HAVE ERASER HANDY! Or do on SCRAP PAPER.

REMEMBER: ALWAYS check for common factor FIRST!

Step 1: Place the factors of ax^2 in the first positions of the 2 sets of parentheses that represent the factors.

Step 2: Place 2 possible factors of c into the second positions of the parentheses (can leave out signs for a moment).

Step 3: Find the outer and inner products of the 2 sets of parentheses (O and I of FOIL)

Step 4: Keep trying different factors until the inner and outer products add or subtract to the middle term (bx). Use a system to make sure you've tried all options (systematically)!

Step 5: Decide on the signs of the second position terms so that they add/subtract to the middle term.

Examples: factor

1. $6x^2 - 19x + 15$

$$(2x - 3)(3x - 5)$$

$$\text{Outside} = (2x)(-5) = -10x$$

$$\text{Inside} = (-3)(3x) = -9x$$

$$\begin{aligned} \text{Outside} + \text{Inside} &= -10x - 9x \\ &= -19x \end{aligned}$$

$$\begin{aligned} \text{Check: FOIL } 6x^2 - 10x - 9x + 15 \\ 6x^2 - 19x + 15 \checkmark \end{aligned}$$

2. $15x^2 + 17x - 42$

$$(3x + 7)(5x - 6)$$

$$\begin{aligned} &35x \\ &-18x \\ \hline 35x - 18x &= 17x \end{aligned}$$

$F =$ factors of $15x^2$
 $L =$ factors of -42

$$\begin{aligned} I + O &= 17x \\ O + I - O & \end{aligned}$$

Try different combinations of outside inside until one works.

• Box Method of factoring Trinomial ($a \neq 1$) (p. 173)

For this method, initially we just write the number factors (not the variables)

Example: $4x^2 + 20x + 9$

Again, first we make sure there isn't a common factor that we could factor out. ~~no~~

Think of factors of the first term and the last term. Write the factors vertically.

Now look at the products of the top and bottom numbers (diagonally). Add/subtract the products. Your goal is that the products will add or subtract to make the middle coefficient. (In this case to equal 20.)

You may end up trying all the combinations of the two sets of factors.. but you can stop if you find the pair that work.

$$\begin{array}{cc} 4 & 3 \\ 1 & 3 \end{array}$$

12 3

$$\begin{array}{cc} 4 & 9 \\ 1 & 1 \end{array}$$

4 9

$$\begin{array}{cc} 2 & 3 \\ 2 & 3 \end{array}$$

6 6

$$\begin{array}{cc} 2 & 9 \\ 2 & 1 \end{array}$$

2 18 $2 + 18 = 20$

As always, you can check your answer by multiplying (FOIL).

$$\begin{aligned} (2x + 9)(2x + 1) \\ = 4x^2 + 2x + 18x + 9 \\ = 4x^2 + 20x + 9 \end{aligned}$$

Example: factor the trinomials and verify your answers. Remember the first step is to factor out the GCF, if possible.

$$3s^2 - 13s - 10$$

$$(3s + 2)(s - 5)$$

$$\begin{array}{cc} 3 & 5 \\ 1 & 2 \end{array}$$

$$\begin{array}{cc} 3 & 2 \\ 1 & 5 \end{array}$$

$$-15 + 2 = -13$$

check
 $3s^2 - 15s + 2s - 10$
 $3s^2 - 13s - 10$

$$6x^2 - 21x + 9$$

$$3(2x^2 - 7x + 3)$$

$$3(2x - 1)(x - 3)$$

$$\begin{array}{cc} 2 & 3 \\ 1 & 1 \end{array}$$

$$\begin{array}{cc} 2 & 1 \\ 1 & 3 \end{array}$$

Check: $3(2x^2 - 6x - x + 3)$
 $3(2x^2 - 7x + 3)$
 $6x^2 - 21x + 9$

P. 166

Homework: #11 the ones you didn't do for AC
(14, 15 more practice)

P. 177 11a, 12, 13 a-c (minimum) #18 a-c
Check for GCF first

3.7 Using the Distributive Property to Multiply Polynomials p. 183

Consider the example: $(2h + 5)(h^2 + 3h - 4)$. Using the distributive property, all terms in the first set of by all terms in the second set of brackets separately. Remember to simplify by combining like terms.

Example: Expand and simplify: $(2h + 5)(h^2 + 3h - 4)$

$$2h^3 + 6h^2 - 8h + 5h^2 + 15h - 20$$

$$2h^3 + 11h^2 + 7h - 20$$

Distribute each term of the first bracket by each term of the second bracket

Combine like terms or simplify

Try it

Examples: $(3b + 4)(b^2 - 2b - 7)$

$$3b^3 - 6b^2 - 21b + 4b^2 - 8b - 28$$

$$3b^3 - 2b^2 - 29b - 28$$

$$(-3f^2 + 3f - 2)(4f^2 - f - 6)$$

$$-12f^4 + 3f^3 + 18f^2 + 12f^3 - 3f^2 - 18f - 8f^2 + 2f + 12$$

$$-12f^4 + 15f^3 + 7f^2 - 16f + 12$$

2: Multiplying Polynomials in More than One Variable (p. 184)

Consider the example: $(x - y)(x^2 + 3xy)$. Using the distributive property, multiply both terms in the first set of brackets (binomial) by all both terms in the second set of brackets (binomial). Remember to simplify by combining like terms.

Example: $(x - y)(x^2 + 3xy)$

$$x^3 + 3x^2y - x^2y - 3xy^2$$

$$= x^3 + 2x^2y - 3xy^2$$

like terms:
same variables
+
exponents

Examples: Expand and simplify:

a) $(4k - 3m)(4k + 3m)$

$$16k^2 - 12mk + 12mk - 9m^2$$

$$16k^2 - 9m^2$$

difference
of
squares
p. 17

b) $(4k - 3m)^2$

$$(4k - 3m)(4k - 3m)$$

$$16k^2 - 12mk + 12mk - 9m^2$$

$$16k^2 - 24mk + 9m^2$$

perfect squares: $\sqrt{16k^2} = 4k$, $\sqrt{9m^2} = 3m$

$$= 2 \cdot (4k)(3m)$$

$$= 24mk$$

c) $(4k + 3m)^2$

pattern:

$$(4k)^2 + 2(4k)(3m) + (3m)^2$$

$$= 16k^2 + 24mk + 9m^2$$

or

$$(4k + 3m)(4k + 3m)$$

$$16k^2 + 12mk + 12mk + 9m^2$$

3. Simplifying Sums and Differences (p. 185)

Consider the example: $(2c - 3)(c + 5) + 3(c - 3)(-3c + 1)$. The problem has added order of operations, therefore, you must apply BEDMAS to solve. *mult then add*

$$\begin{aligned}
 &= 2c^2 + 10c - 3c - 15 + 3(-3c^2 + c + 9c - 3) \\
 &= 2c^2 + 7c - 15 + 3(-3c^2 + 10c - 3) \\
 &= 2c^2 + 7c - 15 - 9c^2 + 30c - 9 \\
 &= -7c^2 + 37c - 24.
 \end{aligned}$$

Try it

Examples: Expand and simplify:

a) $(3x + y - 1)(2x - 4) - (3x + 2y)^2$ b) $(4m + 1)(3m - 2) + 2(2m - 1)(-3m + 4)$

$$\begin{aligned}
 a) &= 6x^2 - 12x + 2xy - 4y - 2x + 4 - (9x^2 + 12xy + 4y) \\
 &= 6x^2 + 2xy - 14x + 4 - 9x^2 - 12xy - 4y \\
 &= -3x^2 - 10xy - 14x - 4y + 4.
 \end{aligned}$$

$$\begin{aligned}
 b) &= 12m^2 - 8m + 3m - 2 + 2(-6m^2 + 8m + 3m - 4) \\
 &= 12m^2 - 5m - 2 + 2(-6m^2 + 11m - 4) \\
 &= 12m^2 - 5m - 2 - 12m^2 + 22m - 8 \\
 &= 17m - 10.
 \end{aligned}$$

Consider the example: $(3x + 4)(x - 5)(2x + 8)$. To solve this problem, you need to multiply the first two binomials, and then simplify. Next apply the distributive property to the product of the first two binomials and the third binomial.

Example: $(3x + 4)(x - 5)(2x + 8)$

$$\begin{aligned}
 &= (3x^2 - 15x + 4x - 20)(2x + 8) \\
 &= (3x^2 - 11x - 20)(2x + 8) \\
 &= 6x^3 + 24x^2 - 22x^2 - 88x - 40x - 160 \\
 &= 6x^3 + 2x^2 - 128x - 160
 \end{aligned}$$

Distribute each term of the first binomial by each term of the second binomial. **Fo/L**

Distribute each term of the third binomial by each term of the product of the first two binomials.

* p. 186 7a i & ii, 7b, 8a & c, 9d, 10a, 11
12 (Volume = area of base \times height)

HOMEWORK: _____

HOMEWORK: page 194 - 195 #11 - 12. Due tomorrow.

13a, 14, 15a, 17

18a 20a 21a & c

Challenge #16

Factoring Special Polynomials

1. Factoring a Perfect Square Trinomial (p. 190) (see also p. 17 of notes)

Reminder: When factoring a trinomial in the form: $ax^2 + bx + c$, where the coefficient (a) is **NOT** equal to 1, the first step is **always** to find the greatest common factor.

Example: Factor the trinomials: $4x^2 + 12x + 9$

$$= (2x+3)(2x+3) \\ = (2x+3)^2$$

$$\begin{array}{c} \uparrow \quad \uparrow \\ \text{perfect} \\ \text{square} \\ 2(\sqrt{4x^2})(\sqrt{9}) \\ = 2 \cdot 2x \cdot 3 \end{array}$$

We call these perfect square trinomials.

$$8x^2 + 40x + 50$$

$$\#1 \text{ GCF} = 2(4x^2 + 20x + 25)$$

$$\#2 \text{ pattern?}$$

$$\begin{array}{c} \uparrow \quad \uparrow \\ \text{perfect} \\ \text{square} \\ 2 \cdot 2x \cdot 5 = 20x \\ \therefore 2(2x+5)^2 \end{array}$$

(They are trinomials (3 terms), and the first and last terms are perfect squares (hence "perfect square trinomial"). The middle term is the product of the square roots of the first and last terms, multiplied by 2. The middle term can be added or subtracted.)

NOW when factoring:

1. Check for GCF (and factor out if possible)
2. Check to see if trinomial is a perfect square trinomial.
3. If not perfect square trinomial, carry on factoring as per usual.

Try these:

a) $4a^2 - 20a + 25$ b) $4 - 20x + 25x^2$ c) $4x^2 + 12x + 9$

$$(2a-5)^2$$

$$\begin{array}{c} 25x^2 - 20x + 4 \\ (5x-2)^2 \end{array}$$

$$(2x+3)^2$$

Question: Why can you only use the shortcut when either all three terms are positive, or when ONLY the second term is negative? Why does the shortcut not work when either the first or third term is a negative???

← 1st + 3rd wouldn't be perfect square

3.8 2. Factoring Trinomials in Two Variables (p. 191)

When factoring a trinomial with two variables, you follow much the same process you would if you were factoring a trinomial with one variable.

Example: Factor the trinomial: $15m^2 + 7mn - 4n^2$

Always try to find the greatest common factor and write the sum as a product.

There is no GCF for $15m^2 + 7mn - 4n^2$

****Check the leading coefficient to decide which method to use in factoring****

Since the leading coefficient is not equal to 1, one of the methods p. 22 or 23.

$$(3m - n)(5m + 4n)$$

$$\begin{array}{r} 3 \quad 4 \quad 3 \quad 1 \\ 5 \quad 1 \quad 5 \quad 4 \end{array}$$

Add in all the variables at the end, once you have figured out the number values of the terms.

Verify your answer:

$$\begin{array}{r} 15m^2 + 12mn - 5mn - 4n^2 \\ 15m^2 + 7mn - 4n^2 \end{array}$$

Example: Factor the trinomial: $x^2 - 3xy + 2y^2$

(Is there a GCF?)

$$(x - y)(x - 2y)$$

****Check the leading coefficient to decide which method to use in factoring****

Since the leading coefficient is equal to 1, use product sum.

Think of the pairs of factors of 2, that add up to -3: $-2 - 1$

check $x^2 - 2xy - xy + 2y^2$

Add in the variables at the end.

4. Factoring a Difference of Squares (p. 193) (and notes p. 17)

Expand the following, and collect like terms:

a) $(x + 3)(x - 3)$

$$x^2 + 3x - 3x - 9$$

$$x^2 - 9$$

b) $(x - 3)(x - 3)$

$$x^2 - 3x - 3x + 9$$

$$x^2 - 6x + 9$$

We call (a) a difference of squares. (It is a binomial - 2 terms. The first and last terms are perfect squares. The terms are subtracted - hence "difference" of squares.)

NOW when factoring:

1. Factor out the GCF if there is one.
2. If it's a binomial - is it a difference of squares?
3. If it's trinomial - is it a perfect square trinomial?
4. If not 2 or 3.. carrying on factoring as per usual.

Example: Factor: $y^2 - 36$

$$= (y - 6)(y + 6)$$

$2x^2 - 2$

$$= 2(x^2 - 1)$$

$$= 2(x - 1)(x + 1)$$

$= 100r^2 - 49s^2$

$$(10r - 7s)(10r + 7s)$$

$3m^2 - 27n^2$

$$= 3(m^2 - 9n^2)$$

$$= 3(m - 3n)(m + 3n)$$

Sometimes, factoring differences of squares could occur more than once in each question.

Example: Factor $x^4 - 16$

NOTE: The second polynomial after initial factoring could also be factored using difference of squares.

$$\begin{aligned} (x^2)(x^2) &= (x^2 - 4)(x^2 + 4) \\ &= x^{2+2} \\ &= x^4 \end{aligned} \quad = (x-2)(x+2)(x^2 + 4)$$

Try these:

Example: $5x^4 - 80$

$$\begin{aligned} &= 5(x^4 - 16) \\ &= 5(x^2 - 4)(x^2 + 4) \\ &= 5(x-2)(x+2)(x^2 + 4) \end{aligned}$$

Example: $162v^4 - 2w^4$

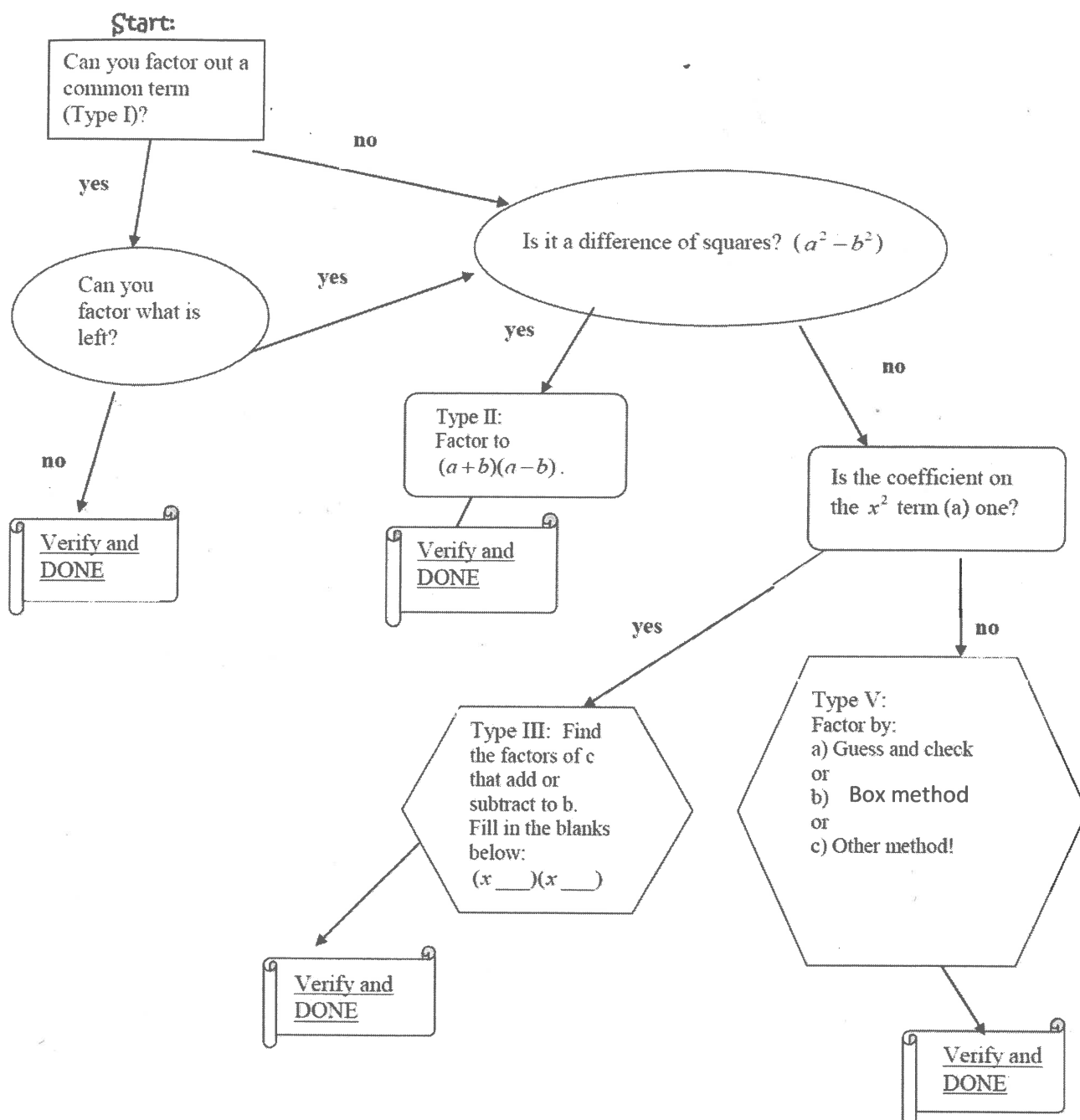
$$\begin{aligned} &= 2(81v^4 - w^4) \\ &= 2(9v^2 - w^2)(9v^2 + w^2) \\ &= 2(3v - w)(3v + w)(9v^2 + w^2) \end{aligned}$$

ONCE YOU HAVE FACTORED, ALWAYS CHECK TO SEE IF IT CAN BE FACTORED FURTHER.

HOMEWORK: _____

Factoring Flow Chart

Quadratics: $ax^2 + bx + c$



Factoring- All Types!!!

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Date _____

Factor. Hint: take out the GCF

1) $5x^5 - 30x^2$
 $5x^2(x^3 - 6)$

2) $-18v^2 - 9v$
 $-9v(2v + 1)$

3) $32x^2y^4 - 24xy$
 $8xy(4xy^3 - 3)$

4) $90a^9 + 27a^2b^3$
 $9a^2(10a^7 + 3b^3)$

5) $20a^9b^5 + 4a^5b^4$
 $4a^5b^4(5a^4b + 1)$

6) $2x^2 + 12x - 10$
 $2(x^2 + 6x - 5)$
 ~~$2(x$~~

7) $4ba^4 - 18b^3 - 8b$
 $2b(2a^4 - 9b^2 - 4)$
 ~~$2b$~~

8) $21xy^5 - 42x^2y + 49xy$
 $7xy(3y^4 - 6x + 7)$

Factor. Hint: Starts with 1 (easy problems!)

9) $a^2 - 5a - 6$
 $(a - 6)(a + 1)$

10) $n^2 + n - 12$
 $(n - 3)(n + 4)$

11) $x^2 - 13x + 36$
 $(x - 9)(x - 4)$

12) $x^2 - 7x + 10$
 $(x - 5)(x - 2)$

Factor.

13) $10r^2 - 11r + 1$
 $(10r - 1)(r - 1)$

14) $4k^2 + 8k - 21$
 $(2k - 3)(2k + 7)$ 33

$$15) 4p^2 - 20p + 9$$

$$(2p-9)(2p-1)$$

not perfect square
trinomials

not $(2p)(3)(2)$

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$$16) 4m^2 - 39m - 10$$

$$(m-10)(4m+1)$$

Factor. Hint: take out a common factor 1st!

$$17) 20x^2 + 82x + 80$$

$$2(10x^2 + 41x + 40)$$

$$2(5x+8)(2x+5)$$

$$18) 20n^2 - 66n - 14$$

$$2(10n^2 - 33n - 7)$$

$$2(2n-7)(5n+1)$$

$$19) 40x^2 + 44x - 24$$

$$4(10x^2 + 11x - 6)$$

$$4(5x-2)(2x+3)$$

$$20) 12p^2 - 26p - 16$$

$$2(6p^2 - 13p - 8)$$

$$2(3p-8)(2p+1)$$

Factor each completely. Hint: different of two squares

$$21) 4v^2 - 25$$

$$(2v-5)(2v+5)$$

$$22) 9b^2 - 4$$

$$(3b-2)(3b+2)$$

$$23) x^2 - 16$$

$$(x-4)(x+4)$$

$$24) 9x^2 - 1$$

$$(3x-1)(3x+1)$$

Factor each completely. Hint: perfect squares *trinomials*

$$25) 9k^2 - 12k + 4$$

$$(3k-2)^2$$

$$26) 25a^2 + 20a + 4$$

$$(5a+2)^2$$

Directions: Factor each expression below completely. Show ALL Work!

<p>8)</p> $196 - 4y^2$ $4(49 - y^2)$ $4(7-y)(7+y)$	<p>9)</p> $2ax - 5x$ $x(2a - 5)$	<p>10)</p> $24a^2b + 18abc$ $6ab(4a + 3c)$
<p>11)</p> $4x^2 - 24x - 28$ $4(x^2 - 6x - 7)$ $4(x - 7)(x + 1)$	<p>12)</p> $49m^2 - 100n^2$ $(7m - 10n)(7m + 10n)$	<p>13)</p> $20 + 9x + x^2$ $x^2 + 9x + 20$ $(x + 5)(x + 4)$
<p>14)</p> $x^4 - 1$ $(x^2 - 1)(x^2 + 1)$ $(x - 1)(x + 1)(x^2 + 1)$	<p>15)</p> $6x^2 - 6y^2$ $6(x^2 - y^2)$ $6(x - y)(x + y)$	<p>16)</p> $2x^3 - 8x$ $2x(x^2 - 4)$
<p>17)</p> $5x^2 + 20x + 20$ $5(x^2 + 4x + 4)$ $5(x + 2)(x + 2)$ $5(x + 2)^2$	<p>18)</p> $5x^3 - 10x^2 - 15x$ $5x(x^2 - 2x - 3)$ $5x(x - 3)(x + 1)$	<p>19)</p> $x^2 - x - 56$ $(x - 8)(x + 7)$